

Relational Algebra

CSI-406 Database Systems



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Relational Algebra

- The basic set of operations for the *relational model* is known as the *relational algebra*.
- These operations enable a user to specify *basic retrieval requests*.
- The result of a retrieval is a *new relation*, which may have been *formed from one or more relations*.

Relational Algebra cont...

- The **algebra operations** thus produce new relations, which can be further manipulated using operations of the same algebra.
- A sequence of relational algebra operations forms a *relational algebra expression*, whose result will also be a relation that represents the result of a *database query* (or retrieval request).

Relational Algebra cont...

Unary Relational Operations (SELECT)

- **SELECT Operation:** SELECT operation is used to select a *subset* of the tuples from a relation that satisfy a *selection condition*.
- *Selection condition* a filter that keeps *only those tuples that satisfy a qualifying condition* – “those satisfying the condition are selected while others are discarded”.

Relational Algebra cont...

Unary Relational Operations (SELECT)

- **Example:** To select the EMPLOYEE tuples whose department number is four or those whose salary is greater than \$30,000 the following notation is used:
- $\sigma_{DNO=4}(\text{EMPLOYEE})$
- $\sigma_{SALARY > 30,000}(\text{EMPLOYEE})$

Relational Algebra cont...

Unary Relational Operations (SELECT)

- In general, the select operation is denoted by σ <selection condition> (**R**) where the symbol σ (**sigma**) is used to denote the *select operator*, and the *selection condition* is a *Boolean expression* specified on the *attributes of relation R*.

Relational Algebra cont...

Unary Relational Operations (SELECT Operation Properties)

- The SELECT operation $\sigma_{\langle \text{selection condition} \rangle} (R)$ produces a relation S that has the **same schema as R**
- The SELECT operation σ is **commutative**; i.e.:
- $\sigma_{\langle \text{condition1} \rangle} (\sigma_{\langle \text{condition2} \rangle} (R)) = \sigma_{\langle \text{condition2} \rangle} (\sigma_{\langle \text{condition1} \rangle} (R))$

Relational Algebra cont...

Unary Relational Operations (SELECT Operation Properties)

- A cascaded SELECT operation **may be applied in any order**; i.e.,
- $\sigma_{\langle \text{condition1} \rangle} (\sigma_{\langle \text{condition2} \rangle} (\sigma_{\langle \text{condition3} \rangle} (R))) = \sigma_{\langle \text{condition2} \rangle} (\sigma_{\langle \text{condition3} \rangle} (\sigma_{\langle \text{condition1} \rangle} (R)))$
- A cascaded SELECT operation may be replaced by a single selection with a conjunction of all the conditions; i.e.,
- $\sigma_{\langle \text{condition1} \rangle} (\sigma_{\langle \text{condition2} \rangle} (\sigma_{\langle \text{condition3} \rangle} (R))) = \sigma_{\langle \text{condition1} \rangle \text{ AND } \langle \text{condition2} \rangle \text{ AND } \langle \text{condition3} \rangle} (R)$

Relational Algebra cont...

Unary Relational Operations (SELECT & PROJECT)

Figure 6.1

Results of SELECT and PROJECT operations. (a) $\sigma_{(Dno=4 \text{ AND } Salary > 25000) \text{ OR } (Dno=5 \text{ AND } Salary > 30000)}$ (EMPLOYEE).
 (b) $\pi_{Lname, Fname, Salary}$ (EMPLOYEE). (c) $\pi_{Sex, Salary}$ (EMPLOYEE).

(a)

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5

(b)

Lname	Fname	Salary
Smith	John	30000
Wong	Franklin	40000
Zelaya	Alicia	25000
Wallace	Jennifer	43000
Narayan	Ramesh	38000
English	Joyce	25000
Jabbar	Ahmad	25000
Borg	James	55000

(c)

Sex	Salary
M	30000
M	40000
F	25000
F	43000
M	38000
M	25000
M	55000

Relational Algebra cont...

Unary Relational Operations (PROJECT)

- This operation *selects certain columns* from the table and *discards the other columns*.
- The PROJECT creates a vertical partitioning – one with the needed columns (attributes) containing results of the operation and other containing the discarded Columns.

Relational Algebra cont...

Unary Relational Operations (PROJECT)

- **Example:** To list each employee's first and last name and salary, the following is used:

- $\pi_{\text{LNAME, FNAME, SALARY}}(\text{EMPLOYEE})$

Relational Algebra cont...

Unary Relational Operations (PROJECT)

- The general form of the project operation is $\pi\langle\text{attribute list}\rangle(\mathbf{R})$ where π (**pi**) is the symbol used to represent the project operation and $\langle\text{attribute list}\rangle$ is the desired list of attributes from the attributes of relation R.
- The project operation *removes any duplicate tuples*, so the result of the project operation is a set of tuples and hence a valid relation.

Relational Algebra cont...

Unary Relational Operations (PROJECT operation properties)

- The number of tuples in the result of projection $\pi_{\langle \text{list} \rangle} (R)$ is always ***less or equal to the number of tuples in R.***
- If the list of attributes includes a key of R, then the number of tuples (in a result of project) is equal to the number of tuples in R.
- $\pi_{\langle \text{list1} \rangle} (\pi_{\langle \text{list2} \rangle} (R)) = \pi_{\langle \text{list1} \rangle} (R)$ as long as $\langle \text{list1} \rangle$ contains the attributes in $\langle \text{list2} \rangle$

Relational Algebra cont...

Unary Relational Operations (PROJECT operation properties)

Figure 6.1

Results of SELECT and PROJECT operations. (a) $\sigma_{(Dno=4 \text{ AND } Salary>25000) \text{ OR } (Dno=5 \text{ AND } Salary>30000)}$ (EMPLOYEE).
(b) $\pi_{Lname, Fname, Salary}$ (EMPLOYEE). (c) $\pi_{Sex, Salary}$ (EMPLOYEE).

(a)

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5

(b)

Lname	Fname	Salary
Smith	John	30000
Wong	Franklin	40000
Zelaya	Alicia	25000
Wallace	Jennifer	43000
Narayan	Ramesh	38000
English	Joyce	25000
Jabbar	Ahmad	25000
Borg	James	55000

(c)

Sex	Salary
M	30000
M	40000
F	25000
F	43000
M	38000
M	25000
M	55000

Relational Algebra cont...

Unary Relational Operations (PROJECT operation properties)

- The number of tuples in the result of projection $\pi_{\langle \text{list} \rangle} (R)$ is always ***less or equal to the number of tuples in R.***
- If the list of attributes includes a key of R, then the number of tuples (in a result of project) is equal to the number of tuples in R.
- $\pi_{\langle \text{list1} \rangle} (\pi_{\langle \text{list2} \rangle} (R)) = \pi_{\langle \text{list1} \rangle} (R)$ as long as $\langle \text{list1} \rangle$ contains the attributes in $\langle \text{list2} \rangle$

Relational Algebra cont...

Unary Relational Operations (Rename)

- We may want to apply several relational algebra operations one after the other.
- Either we can write the operations as a single **relational algebra expression** by nesting the operations, or we can apply one operation at a time and create **intermediate result relations**.

Relational Algebra cont...

Unary Relational Operations (Rename)

- In the latter case, we must give names to the relations that hold the intermediate results.
- **Example:** To retrieve the *first name*, *last name*, and *salary* of all employees who work in *department number 5*, we must apply a select and a project operation. We can write a single relational algebra expression as follows: $\pi_{\text{FNAME, LNAME, SALARY}}(\sigma_{\text{DNO}=5}(\text{EMPLOYEE}))$

Relational Algebra cont...

Unary Relational Operations (Rename)

- OR, we can explicitly show the sequence of operations, giving a name to each intermediate relation:
- **DEP5_EMPS** $\leftarrow \sigma_{DNO=5}(\text{EMPLOYEE})$
- **RESULT** $\leftarrow \pi_{FNAME, LNAME, SALARY}(\text{DEP5_EMPS})$

Relational Algebra cont...

Unary Relational Operations (Rename)

- The rename operator is ρ
- The general Rename operation can be expressed by any of the following forms:
- $\rho_S(B_1, B_2, \dots, B_n)(R)$ is a renamed relation **S** based on **R** with column names B_1, B_1, \dots, B_n .

Relational Algebra cont...

Unary Relational Operations (Rename)

- $\rho_S (R)$ is a renamed relation S based on R (which does not specify column names).
- $\rho_{(B_1, B_2, \dots, B_n)} (R)$ is a renamed relation with column names B_1, B_2, \dots, B_n which does not specify a new relation name.

Relational Algebra cont...

Unary Relational Operations (Rename)

(a)

Fname	Lname	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

Figure 6.2

Results of a sequence of operations. (a) $\pi_{Fname, Lname, Salary}(\sigma_{Dno=5}(EMPLOYEE))$. (b) Using intermediate relations and renaming of attributes.

(b)

TEMP

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

R

First_name	Last_name	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

Relational Algebra cont...

Operations From Set Theory (Union)

- The result of this operation, denoted by $R \cup S$, is a relation that **includes all tuples that are either in R or in S or in both R and S.**
- Duplicate tuples are eliminated.
- **Example:** To retrieve the social security numbers of all employees who either *work in department 5* or *directly supervise an employee who works in department 5*, we can use the union operation as follows:

Relational Algebra cont...

Operations From Set Theory (Union)

- **DEP5_EMPS** $\leftarrow \sigma_{DNO=5}(\text{EMPLOYEE})$
- **RESULT1** $\leftarrow \pi_{SSN}(\text{DEP5_EMPS})$
- **RESULT2 (SSN)** $\leftarrow \pi_{SUPERSSN}(\text{DEP5_EMPS})$
- **RESULT** $\leftarrow \text{RESULT1} \cup \text{RESULT2}$

Fname	Minit	Lname	<u>Ssn</u>	Bdate	Address	Sex	Salary	Super_ssn	Dno
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5

Relational Algebra cont...

Operations From Set Theory (Union)

- As a single relational algebra expression, this becomes:
- **Result** ← $\pi_{Ssn} (\sigma_{Dno=5} (\text{EMPLOYEE})) \cup \pi_{Super_ssn} (\sigma_{Dno=5} (\text{EMPLOYEE}))$
- The union operation produces the tuples that are in either RESULT1 or RESULT2 or both. The two operands must be “type compatible”.

Relational Algebra cont...

Operations From Set Theory (Union)

RESULT1

Ssn
123456789
333445555
666884444
453453453

RESULT2

Ssn
333445555
888665555

RESULT

Ssn
123456789
333445555
666884444
453453453
888665555

Figure 6.3

Result of the UNION operation
 $RESULT \leftarrow RESULT1 \cup RESULT2$.

Relational Algebra cont...

Operations From Set Theory (Type Compatibility)

- The operand relations $R_1(A_1, A_2, \dots, A_n)$ and $R_2(B_1, B_2, \dots, B_n)$ must have the *same number of attributes*, and the *domains of corresponding attributes must be compatible*; that is, $\text{dom}(A_i) = \text{dom}(B_i)$ for $i=1, 2, \dots, n$.
- The resulting relation for $R_1 \cup R_2$, $R_1 \cap R_2$, or $R_1 - R_2$ has the same attribute names as the *first* operand relation R_1 (by convention).

Relational Algebra cont...

Operations From Set Theory (Type Compatibility)

Figure 6.4

The set operations UNION, INTERSECTION, and MINUS. (a) Two union-compatible relations. (b) $STUDENT \cup INSTRUCTOR$.

(a) **STUDENT**

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

INSTRUCTOR

Fname	Lname
John	Smith
Ricardo	Browne
Susan	Yao
Francis	Johnson
Ramesh	Shah

(b)

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert
John	Smith
Ricardo	Browne
Francis	Johnson

- **UNION Example** $STUDENT \cup INSTRUCTOR$

Relational Algebra cont...

Operations From Set Theory (Type Compatibility)

- The operand relations $R_1(A_1, A_2, \dots, A_n)$ and $R_2(B_1, B_2, \dots, B_n)$ must have the *same number of attributes*, and the *domains of corresponding attributes must be compatible*; that is, $\text{dom}(A_i) = \text{dom}(B_i)$ for $i=1, 2, \dots, n$.
- The resulting relation for $R_1 \cup R_2$, $R_1 \cap R_2$, or $R_1 - R_2$ has the same attribute names as the *first* operand relation R_1 (by convention).

Relational Algebra cont...

Operations From Set Theory (Intersection)

- The result of this operation, denoted by $\mathbf{R} \cap \mathbf{S}$, is a relation that includes all tuples that are **in both R and S**.
- The two **operands** must be "**type compatible**"
- **Example:** The result of the intersection operation (figure below) includes only those who are both students and instructors.

Relational Algebra cont...

Operations From Set Theory (Intersection)

Figure 6.4

The set operations UNION, INTERSECTION, and MINUS. (a) Two union-compatible relations. (c) $STUDENT \cap INSTRUCTOR$.

(a) **STUDENT**

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

INSTRUCTOR

Fname	Lname
John	Smith
Ricardo	Browne
Susan	Yao
Francis	Johnson
Ramesh	Shah

(c)

Fn	Ln
Susan	Yao
Ramesh	Shah

- **INTERSECTION Example** $STUDENT \cap INSTRUCTOR$

Relational Algebra cont...

Operations From Set Theory (MINUS)

- MINUS operation, also called **Set Difference Operation**
- The result of this operation, denoted by $\mathbf{R - S}$, is a relation that includes all tuples that are **in R but not in S**.
- The two **operands** must be "**type compatible**".
- **Example:** The figure shows the names of students who are not instructors, and the names of instructors who are not students.

Relational Algebra cont...

Operations From Set Theory (MINUS)

Figure 6.4

The set operations UNION, INTERSECTION, and MINUS. (a) Two union-compatible relations. (d) STUDENT – INSTRUCTOR. (e) INSTRUCTOR – STUDENT.

(a) STUDENT

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

INSTRUCTOR

Fname	Lname
John	Smith
Ricardo	Browne
Susan	Yao
Francis	Johnson
Ramesh	Shah

(d)

Fn	Ln
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

(e)

Fname	Lname
John	Smith
Ricardo	Browne
Francis	Johnson

Set Difference (or MINUS) Operation Example

D = STUDENT – INSTRUCTOR

E = INSTRUCTOR – STUDENT

Relational Algebra cont...

Operations From Set Theory

Figure 6.4

The set operations UNION, INTERSECTION, and MINUS. (a) Two union-compatible relations. (b) $STUDENT \cup INSTRUCTOR$. (c) $STUDENT \cap INSTRUCTOR$. (d) $STUDENT - INSTRUCTOR$. (e) $INSTRUCTOR - STUDENT$.

(a) STUDENT

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

INSTRUCTOR

Fname	Lname
John	Smith
Ricardo	Browne
Susan	Yao
Francis	Johnson
Ramesh	Shah

(b)

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert
John	Smith
Ricardo	Browne
Francis	Johnson

(c)

Fn	Ln
Susan	Yao
Ramesh	Shah

(d)

Fn	Ln
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

(e)

Fname	Lname
John	Smith
Ricardo	Browne
Francis	Johnson

Relational Algebra cont...

Operations From Set Theory

- Notice that both union and intersection are *commutative operations*; that is:
 - $R \cup S = S \cup R$, and $R \cap S = S \cap R$
 - The minus operation is *not commutative*; that is, in general:
 - $R - S \neq S - R$

Relational Algebra cont...

Operations From Set Theory

- Both union and intersection can be treated as n-ary operations applicable to any number of relations as both are *associative operations*; that is:
- **$R \cup (S \cup T) = (R \cup S) \cup T$, and $(R \cap S) \cap T = R \cap (S \cap T)$**