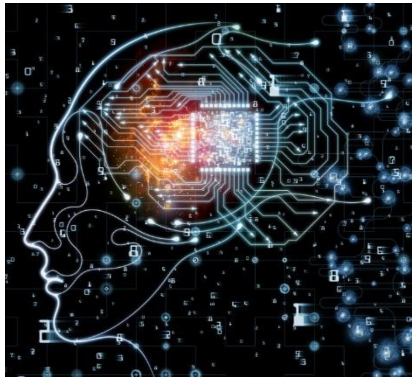
Boolean Algebra

Digital Logic and Design



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Course Outline

Digital Logic Design
Course Contents:
Number Systems, Logic Gates, Boolean Algebra, Combination logic circuits and designs, Simplification Methods (K-Map, Quinn Mc-Cluskey method), Flip Flops and Latches, Asynchronous and Synchronous circuits, Counters, Shift Registers, Counters, Triggered devices & its types. Binary Arithmetic and Arithmetic Circuits, Memory Elements, State Machines. Introduction Programmable Logic Devices (CPLD, FPGA); Lab Assignments using tools such as Verilog HDL/VHDL, MultiSim
Reference Material:
1. Digital Fundamentals by Floyd, 11/e. 2. Fundamental of Digital Logic with Verilog Design, Stephen Brown, 2/e.

Postulates and Theorems

- The postulates are basic axioms of the algebraic structure and need no proof.
- The theorems must be proven from the postulates.

Duality

- Some postulates are listed here in pairs and designated by part (a) and part (b):

1. (a) The structure is closed with respect to the operator $+$.
(b) The structure is closed with respect to the operator $\cdot$.

2. (a) The element 0 is an identity element with respect to $+$; that is, $x + 0 = 0 + x = x$.
(b) The element 1 is an identity element with respect to $\cdot$; that is, $x \cdot 1 = 1 \cdot x = x$.

Duality cont...

3. (a) The structure is commutative with respect to $+$; that is, $x + y = y + x$.
(b) The structure is commutative with respect to $\cdot$; that is, $x \cdot y = y \cdot x$.

4. (a) The operator $\cdot$ is distributive over $+$; that is, $x \cdot (y + z) = (x \cdot y) + (x \cdot z)$.
(b) The operator $+$ is distributive over $\cdot$; that is, $x + (y \cdot z) = (x + y) \cdot (x + z)$.

5. For every element $x \in B$, there exists an element $x' \in B$ (called the *complement* of x) such that (a) $x + x' = 1$ and (b) $x \cdot x' = 0$.

6. There exist at least two elements $x, y \in B$ such that $x \neq y$.

Duality cont…

- One part may be obtained from the other if the **binary operators** and the **identity elements** are ***<u>interchanged</u>***.
- If the *dual* of an algebraic expression is desired, we simply interchange **OR and AND** operators and replace **1's by 0's** and **0's by 1's**.

Duality cont…

- This important property of Boolean algebra is called the ***duality principle*** and states that:
- "every algebraic expression deducible from the postulates of Boolean algebra remains valid if the operators and identity elements are interchanged"

Duality cont…

Postulate 2	(a)	$x + 0 = x$	(b)	$x \cdot 1 = x$
Postulate 5	(a)	$x + x' = 1$	(b)	$x \cdot x' = 0$
Theorem 1	(a)	$x + x = x$	(b)	$x \cdot x = x$
Theorem 2	(a)	$x + 1 = 1$	(b)	$x \cdot 0 = 0$
Theorem 3, involution		$(x')' = x$		
Postulate 3, commutative	(a)	$x + y = y + x$	(b)	$xy = yx$
Theorem 4, associative	(a)	$x + (y + z) = (x + y) + z$	(b)	$x(yz) = (xy)z$
Postulate 4, distributive	(a)	$x(y + z) = xy + xz$	(b)	$x + yz = (x + y)(x + z)$
Theorem 5, DeMorgan	(a)	$(x + y)' = x'y'$	(b)	$(xy)' = x' + y'$
Theorem 6, absorption	(a)	$x + xy = x$	(b)	$x(x + y) = x$

Theorem Proof

$$x + x = x.$$

Statement	Justification
$x + x = (x + x) \cdot 1$	postulate 2(b)
$= (x + x)(x + x')$	5(a)
$= x + xx'$	4(b)
$= x + 0$	5(b)
$= x$	2(a)

Theorem Proof cont...

$x \cdot x = x.$

Statement	Justification
$x \cdot x = xx + 0$	postulate 2(a)
$= xx + xx'$	5(b)
$= x(x + x')$	4(a)
$= x \cdot 1$	5(a)
$= x$	2(b)

Theorem Proof cont...

$x + 1 = 1.$

$x \cdot 0 = 0$ by duality.

Statement	Justification
$x + 1 = 1 \cdot (x + 1)$	postulate 2(b)
$= (x + x')(x + 1)$	5(a)
$= x + x' \cdot 1$	4(b)
$= x + x'$	2(b)
$= 1$	5(a)

Theorem Proof cont...

$$x + xy = x.$$

Statement	Justification
$x + xy = x \cdot 1 + xy$	postulate 2(b)
$= x(1 + y)$	4(a)
$= x(y + 1)$	3(a)
$= x \cdot 1$	2(a)
$= x$	2(b)

Theorem Proof cont…

- The other way to proof a theorem is a **truth table**:

x	y
0	0
0	1
1	0
1	1

xy	$x + xy$
0	0
0	0
0	1
1	1

Theorem Proof cont…

- Tthe truth table for the first DeMorgan's theorem:

x	y	$x + y$	$(x + y)'$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

x'	y'	$x'y'$
1	1	1
1	0	0
0	1	0
0	0	0

Theorem Proof cont…

- Tthe truth table for the first DeMorgan's theorem:

x	y	$x + y$	$(x + y)'$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

x'	y'	$x'y'$
1	1	1
1	0	0
0	1	0
0	0	0