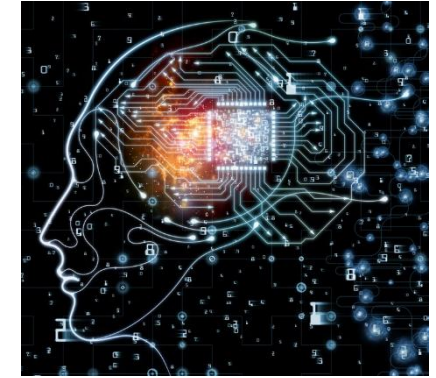


# Simplification Method K-MAP

Digital Logic and Design



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# Course Outline

## Digital Logic Design

### Course Contents:

Number Systems, Logic Gates, Boolean Algebra, Combination logic circuits and designs, Simplification Methods (K-Map, Quinn Mc-Cluskey method), Flip Flops and Latches, Asynchronous and Synchronous circuits, Counters, Shift Registers, Counters, Triggered devices & its types. Binary Arithmetic and Arithmetic Circuits, Memory Elements, State Machines. Introduction Programmable Logic Devices (CPLD, FPGA); Lab Assignments using tools such as Verilog HDL/VHDL, MultiSim

### Reference Material:

1. Digital Fundamentals by Floyd, 11/e.
2. Fundamental of Digital Logic with Verilog Design, Stephen Brown, 2/e.

# Karnaugh Map

- **Karnaugh Map:** A graphical technique for *simplifying an expression* into a *minimal sum of products* (MSP) form:
  1. There are a minimal number of *product terms* in the expression
  2. Each term has a *minimal number of literals*
- K-map provides a systematic method which is used for simplifying 2, 3, 4 and 5 variable expressions

# Karnaugh Map cont...

## Simplification with postulates

- Simplification of Boolean Expressions by using laws and postulates has some limitations:
  1. Doesn't guarantee simplest form of expression
  2. Terms are not obvious
  3. Special skills of applying rules, laws and postulates are required

# Karnaugh Map cont...

## K Map Reduction Mechanism

- There are three steps to reduce a function by using K Map:
  1. Mapping of function
  2. Grouping
  3. Reduction

Minterms				
x	y	z	Term	Designation
0	0	0	$x'y'z'$	$m_0$
0	0	1	$x'y'z$	$m_1$
0	1	0	$x'yz'$	$m_2$
0	1	1	$x'yz$	$m_3$
1	0	0	$xy'z'$	$m_4$
1	0	1	$xy'z$	$m_5$
1	1	0	$xyz'$	$m_6$
1	1	1	$xyz$	$m_7$

# Karnaugh Map cont...

## K-map with 3 variables

- Used for simplifying 3-variable expressions
- K-map has 8 cells
- K-map can be represented in row format or column format

AB\C	0	1
00	0	1
01	2	3
11	6	7
10	4	5

A\BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

# Karnaugh Map cont...

## K-map with 4 variables

x	y	z	Minterms	
			Term	Designation
0	0	0	$x'y'z'$	$m_0$
0	0	1	$x'y'z$	$m_1$
0	1	0	$x'yz'$	$m_2$
0	1	1	$x'yz$	$m_3$
1	0	0	$xy'z'$	$m_4$
1	0	1	$xy'z$	$m_5$
1	1	0	$xyz'$	$m_6$
1	1	1	$xyz$	$m_7$

- Used for simplifying 4-variable expressions
- K-map has 16 cells
- A 4-variable K-map has a square format

AB\CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

# Karnaugh Map cont...

## Mapping of Standard SOP expression

- Selecting n-variable K-map
  1. Marked with 1 in cell for each minterm
  2. Remaining cells marked with 0



# Karnaugh Map cont...

## Mapping of Standard SOP expression

- SOP expression:  $ABC + \overline{A}BC + A\overline{B}C$
- The cells representing the three minterms are marked with 1s, remaining cells are marked with 0s.
- Any of the two K-maps can be used

A\BC	00	01	11	10
0	0	0	0	1
1	1	0	0	1

AB\C	0	1
00	0	0
01	1	0
11	1	0
10	1	0

# Karnaugh Map cont...

## Mapping of Standard SOP expression

- SOP expression:

$$\overline{A}\overline{B}\overline{C}.D + \overline{A}.B.\overline{C}.\overline{D} + \overline{A}.B.\overline{C}.D + \overline{A}.B.C.\overline{D} + A.\overline{B}.\overline{C}.\overline{D} + A.B.\overline{C}.D + A.B.C.\overline{D}$$

AB\CD	00	01	11	10
00	0	1	0	0
01	1	1	0	1
11	0	1	0	1
10	1	0	0	0

# Karnaugh Map cont...

## Mapping of Non-Standard SOP expression

- Selecting n-variable K-map
  1. Marked with 1 in all the cells where the non- standard product term is present
  2. Remaining cells marked with 0

# Karnaugh Map cont...

## Mapping of Non-Standard SOP expression

- SOP expression:  $A + B\bar{C}$
- Complete the terms carefully by considering number of variables.
- $ABC + AB'C + ABC' + AB'C'$  (Complete terms for “ A ”)
- $ABC' + A'BC'$  (Complete terms for “ BC' ”)
- If 1 variable is missing then two new terms will be formulated.
- If 2 then 4. If 3 then 8 and so on.... [formula  $2^n$ ]

# Karnaugh Map cont...

## Mapping of Non-Standard SOP expression

- SOP expression:  $A + B\bar{C}$

A\BC	00	01	11	10
0				
1				

AB\C	0	1
00	0	0
01	1	0
11	1	1
10	1	1

$$ABC + AB'C + ABC' + AB'C'$$

$$ABC' + A'BC'$$

A\BC	00	01	11	10
0	0	0	0	1
1	1	1	1	1

# Karnaugh Map cont...

## Mapping of Non-Standard SOP expression

- SOP expression:  $D + A\bar{C} + BC$

AB\CD	00	01	11	10
00				
01				
11				
10				

# Karnaugh Map cont...

## Mapping of Non-Standard SOP expression

- SOP expression:  $D + A\bar{C} + BC$

AB\CD	00	01	11	10
00	0	1	1	0
01	0	1	1	1
11	1	1	1	1
10	1	1	1	0

# Karnaugh Map cont...

## Grouping

AB\CD	00	01	11	10
00				
01				
11				
10				

- To make groups, we have to follow these rules:
  1. K-map is considered to be wrapped around
  2. All sides are adjacent to each other
  3. Groups of 2, 4, 8, 16 and 32 adjacent cells are formed
  4. Groups can be row, column, square or rectangular.
  5. Groups of diagonal cells are not allowed



# Karnaugh Map cont...

## Simplification of SOP expressions using K-map

- Mapping of expression → Forming of Groups of 1s
- Each group represents product term
- 3-variable K-map:
  - 1 cell group yields a **3 variable product term**
  - 2 cell group yields a **2 variable product term**
  - 4 cell group yields a **1 variable product term**
  - 8 cell group yields a **value of 1 for function**

# Karnaugh Map cont...

## Simplification of SOP expressions using K-map

- 4-variable K-map:
- 1 cell group yields a **4 variable product term**
- 2 cell group yields a **3 variable product term**
- 4 cell group yields a **2 variable product term**
- 8 cell group yields a **1 variable product term**
- 16 cell group yields a **value of 1 for function**

# Karnaugh Map cont...

## Simplification of SOP expressions using K-map

$$B.\bar{C} + A.C + \bar{B}.C$$

AB\C	0	1
00	0	1
01	1	0
11	1	1
10	0	1

A\BC	00	01	11	10
0	0	1	1	1
1	1	0	0	0

$$A.\bar{B}.\bar{C} + \bar{A}.C + \bar{A}.B$$

# Karnaugh Map *cont...*

## Simplification of SOP expressions using K-map

$$B + A.C$$

AB\C	0	1
00	0	0
01	1	1
11	1	1
10	0	1

A\BC	00	01	11	10
0	0	0	1	1
1	1	1	1	0

$$A.\bar{B} + B.C + \bar{A}.B$$

# Karnaugh Map *cont...*

## Simplification of SOP expressions using K-map

$$A.\bar{C}.\bar{D} + C.D + B.C$$

AB\CD	00	01	11	10
00	0	0	1	0
01	0	0	1	1
11	1	0	1	1
10	1	0	1	0

# Karnaugh Map *cont...*

## Simplification of SOP expressions using K-map

$$A.\bar{C} + \bar{B}.D + B.C$$

AB\CD	00	01	11	10
00	0	1	1	0
01	0	0	1	1
11	1	1	1	1
10	1	1	1	0

# Karnaugh Map *cont...*

## Simplification of SOP expressions using K-map

$$\bar{B}\bar{D} + \bar{B}C + A.B.D + \bar{A}.C.\bar{D}$$

AB\CD	00	01	11	10
00	1	0	1	1
01	0	0	0	1
11	0	1	1	0
10	1	0	1	1